

Key Characteristics of Effective Numeracy Teaching P-6

Differentiating support for all students



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Contents

Introduction.....	3
Suggested Readings	5
Prep.....	6
Teacher knowledge.....	6
Numeracy focus	6
Assessment	7
Planning and instruction.....	8
Years 1 and 2.....	9
Teacher knowledge.....	9
Numeracy focus	9
Assessment	10
Planning and instruction.....	11
Years 3 and 4.....	12
Teacher knowledge.....	12
Numeracy focus	13
Assessment	14
Planning and instruction.....	15
Years 5 and 6.....	15
Teacher knowledge.....	15
Numeracy focus	16
Assessment	17
Planning and instruction.....	18
Bibliography.....	19

Introduction

The international evidence-base has reached consensus about the significant influence teachers have in the classroom and the knowledge and the skill base required to teach all students well. In order to improve teacher practice, teachers must not only understand what it looks like to improve in different domains of learning but they must also believe they are capable of improving their practice. (e⁵ Instructional Model, DEECD, 2009)

They Key Characteristics of Effective Numeracy Teaching P – 6 provides a framework to support improvement in teaching and learning. The purpose of the document is to:

- Articulate effective practice in numeracy teaching that supports differentiation within the classroom
- Build knowledge and capacity in numeracy teaching and learning with a focus on student improvement
- Establish a common, shared language to describe effective practice in numeracy teaching

The document is applicable to a range of audiences:

- Classroom teachers: to enable them to guide their planning and instruction, based on student assessment information; to build discipline and pedagogical content knowledge; to focus discussion in professional learning teams
- Principals with the leadership team: to enable substantive conversations to take place with teachers on effective practice in numeracy teaching, to inform Strategic and Annual Implementation Plans including professional learning opportunities for the whole staff, teams of teachers and individual teachers
- Regional Network Leaders and other regional staff: to support articulation of clear and consistent messages on numeracy teaching and learning

Students within each class have a wide range of numeracy experiences, abilities and backgrounds. This provides a significant challenge as teachers support all students to develop the knowledge and skills for continuous engagement and improvement in numeracy.

International research indicates that teachers need to consider systematically ‘two phases of teaching’ to respond effectively to the diversity of student learning needs and aspirations at all stages of learning:

- The **first** phase – quality differentiated classroom teaching for all students, and
- The **second** phase – additional short-term intervention to accelerate learning for students not achieving the expected level in numeracy.

The Key Characteristics of Effective Numeracy Teaching specifies first phase, quality differentiated classroom teaching for all students. This requires that ***the teacher proactively plans and carries out varied approaches to content, process, and product in anticipation of and response to student differences in readiness, interest, and learning needs*** (e⁵ Instructional Model, DEECD, 2009).

To plan effectively for differentiated teaching, teachers draw on their knowledge and understanding of key theories and knowledge of the Victorian Essential Learning Standards (VELS) Mathematics standards. This is important for responsive and focused teaching for students at all levels. Teachers also require knowledge of their students, including their interests and prior knowledge, English language proficiency, and identified learning strengths and areas for improvement.

Teachers identify a specific focus for teaching using assessment data. This includes using assessment information to determine purposeful teaching strategies that meet the identified needs of individual students.

The use the e⁵ Instructional Model informs the delivery of the planned curriculum. Teachers assess and monitor students’ progress, adjusting their instruction to meet the needs of students when appropriate.

The Key Characteristics of Effective Numeracy Teaching P-6 is a companion document to the *Victorian Literacy and Numeracy Statement* and the other three *Key Characteristics of Effective Literacy Teaching P-6 and 7-10* and *Key Characteristics of Effective Numeracy Teaching 7-10*. These documents were informed by research and developed through consultation with classroom practitioners, literacy and numeracy experts, coaches and regional personnel.

The document has sections for Prep, Years 1 and 2, Years 3 and 4 and Years 5 and 6, each containing:

- **Teacher knowledge** – disciplinary and pedagogical content knowledge teachers require, at and beyond the level they teach, including VELS Mathematics standards and progression points, Mathematics Developmental Continuum P-10, Early Years Learning and Developmental Framework: Birth to 8 years and information on particular cohorts of students.
- **Numeracy focus** – advice on essential knowledge and skills on which teachers need to focus.
- **Assessment** – advice on key assessments and their timing to inform teaching including Mathematics Online Interview, Fractions and Decimals Online Interview: Victorian Curriculum and Assessment Authority (VCAA) On Demand and Progress Tests, Scaffolding Numeracy in the Middle Years, National Assessment Program – Literacy and Numeracy (NAPLAN) tests.
- **Planning and instruction** – This section refers to the e⁵ Instructional Model which provides a framework to support differentiated, purposeful teaching. It also provides advice on organisational structures and recommended teaching strategies.

Suggested Readings

- [Mathematics domain page](#)

Access information on the Victorian Essential Learning Standards and Mathematics Development Continuum P–10 plus support for numeracy teachers including research, digital learning resources, assessment maps and sample tasks, web links, professional learning tools and other numeracy resources.

- [Research eLert – Numeracy in practice: teaching, learning and using mathematics \(2009\)](#)

Report focusing on research and links related to the characteristics of effective numeracy teaching and addressing key classroom issues including: what to teach, how to teach numeracy, how to cater for diversity and how to make best use of technology.

- [Researching Numeracy Teaching Approaches in Primary Schools \(2002-2003\)](#)

Research identifying twelve scaffolding practices that contribute to improved student learning outcomes and describing a range of communicative practices that teachers use to support students' mathematics learning.

- [Early Numeracy Research Project \(ENRP\)](#)

A three-year (1999-2001) Prep to Year 2 research project, involving 28 primary schools in developing a comprehensive approach to mathematics.

- [Middle Years Numeracy Research Project \(MYNRP\)](#)

Commissioned to inform the development of a strategic and coordinated approach to the teaching and learning of numeracy for students in Years 5–9, the final report for this research was published in 2001.

Prep

Teacher knowledge

Effective teachers require:

- thorough knowledge of VELS mathematics domain [learning focus statements and standards](#) and progression points – levels [1](#) and [2](#) to support planning for differentiated teaching
- knowledge of the [Victorian Early Years Learning and Development Framework: Birth to 8 years](#)
- knowledge of the [Mathematics Developmental Continuum P–10](#), the indicators of progress, teaching strategies and activities within the Continuum
- knowledge of key mathematical concepts in levels 1–2 of the [Developmental Overviews](#)
- knowledge of the [points of growth](#) in the Mathematics Online Interview
- knowledge of the particular needs that students may have in relation to English language and numeracy, including students from [Koorie](#), [ESL](#) and [low SES](#) backgrounds.

Numeracy focus

Effective teachers determine the numeracy focus by referencing the VELS mathematics standards and progression points.

For students achieving at the expected level, effective teachers:

- teach the counting of numbers (from 1–10, 1–20) moving from rote counting to [one-to-one correspondence](#) and making links between counting numbers (cardinal numbers) and placing objects in order ([ordinal numbers](#))
- engage students in identifying and creating different representations of numbers (1–10, 1–20), recognising the concept of zero as a place holder
- teach students to read and write numbers and to count, sort, order and label so they can identify when two sets are equal in size and one set is larger than another and to recognise that [counting a collection \(0–20\)](#) again will always produce the same result no matter how the numbers in the collection are changed or manipulated (trusting the count/conservation of number)
- engage students in developing concepts about number without having to make or count the numbers (subitising) and in [exploring patterns](#) in numbers and space by manipulating objects according to simple rules
- engage students in grouping together and moving apart given numbers, moving to addition and subtraction of the numbers 0–20 and teach ways of recording computations and use simple functions on a calculator
- engage students in identifying and representing points, lines and curves, 2-D and 3-D shapes, including interior and exterior views, moving from recognising and visualising to [comparing, sorting and matching shapes](#)
- teach students to [describe the features, location and orientation](#) of shapes and objects
- teach the vocabulary and conventions when comparing measurements and engage students in tasks using informal units to [measure the length](#), area, capacity and mass of familiar objects, and tasks to [establish awareness of time](#) such as the cycle of days in the week, hours in the day and seasons

For students achieving above the expected level, teachers should refer to the appropriate [VELS standards and progression points](#).

For students needing additional assistance, teachers should refer to the [Victorian Early Years Learning and Development Framework: Birth to 8 years](#).

Assessment

Effective teachers continuously monitor and track the progress of individual students.

At the beginning of the year, effective teachers:

- analyse [Transition Learning and Development Statements](#) for all students
- administer and analyse the [Mathematics Online Interview](#) and refer to the [Observation Notes: Detour for students starting the first year of school](#).

Throughout the year, effective teachers:

- schedule and document ongoing assessment to track individual student's progress
- provide students with regular opportunities for self assessment and self reflection
- administer and analyse the Assessment for Common Misunderstandings for identified students. This assessment task addresses a key area of number:
 - [LEVEL 1 – Trusting the Count, developing flexible mental objects for the numbers 0 to 10](#)
- use the [VCAA Mathematics assessment maps](#) to help moderate student work
- provide timely and frequent feedback on assessment tasks.

At the end of each semester, effective teachers:

- integrate evidence collected throughout the semester to make on-balance judgements against VELS mathematics standards and progression points.

Planning and instruction

Assessment data is the starting point for curriculum planning and differentiated instruction.

Effective teachers:

- demonstrate capabilities as described in the [e⁵ Instructional Model](#)
- dedicate a daily one hour numeracy block (as a minimum) to explicit teaching, targeting important mathematical ideas and making the focus clear
- use a range of flexible student groupings to provide the appropriate level of differentiated teacher support, including whole class focus, small groups, independent activities and whole class reflection
- provide independent time so students can practise what they know and act on reflection and feedback, giving opportunities to make knowledge and skills automatic
- develop mathematical language by explicitly introducing new terms and symbols and expecting and encouraging correct use, making connections between language, symbols and materials
- provide opportunities and resources for students to manipulate concrete materials
- structure purposeful, authentic tasks that allow different possibilities, strategies and products to emerge and encourage higher order thinking skills
- develop numeracy understanding through strategic questioning by the teacher and feedback and explanation of reasoning and methods by the student.

Recommended teaching strategies

Effective teachers:

- explicitly teach students [strategies to approach problems](#)
- select appropriate teaching strategies after analysis of the [Mathematics Online Interview](#)
- engage students in discussion, reflection and active construction throughout sessions to extend their thinking by building on their contributions and questions and to resolve misconceptions
- use a range of practices selected from the [twelve scaffolding practices](#). These are used throughout the five phases of instruction as described in the [e⁵ Instructional Model](#).

Selecting materials

Effective teachers select a range of materials that are:

- based on the students' developmental stages and the mathematics being explored
- appropriate to the learning context
- socially and culturally inclusive.

Years 1 and 2

Teacher knowledge

Effective teachers require:

- thorough knowledge of VELS mathematics domain [learning focus statements and standards](#) and progression points – levels [1](#), [2](#) and [3](#) to support planning for differentiated teaching
- knowledge of the [Victorian Early Years Learning and Development Framework: Birth to 8 years](#)
- knowledge of the [Mathematics Developmental Continuum P–10](#), the indicators of progress, teaching strategies and activities within the Continuum
- knowledge of key mathematical concepts in levels 1–3 of the [Developmental Overviews](#)
- knowledge of the [points of growth](#) in the Mathematics Online Interview
- knowledge of the particular needs that students may have in relation to English language and numeracy, including students from [Koorie](#), [ESL](#) and [low SES](#) backgrounds.

Numeracy focus

Effective teachers determine the numeracy focus by referencing the VELS mathematics standards and progression points.

For students achieving at the expected level, effective teachers:

- teach students to [model, represent and order numbers](#) (0–1000), count forward and backward by 1s, 10s and 100s and [skip counting](#) by 2s, 4s and 5s from 0–100
- teach students to recognise and form patterns, strengthen their visual images for mental computation through [use hundreds charts](#) and number lines
- teach students to [add and subtract](#) one and two digit numbers, recording and mentally, by [counting on](#) and counting back (0–100) and using [number facts](#) (0–20) such as doubles, near doubles and [make-to-10](#)
- teach students to calculate simple multiplications as repeated addition and to recognise that in adding or multiplying numbers (1–100) the order of the numbers will not change the answer
- teach students to [use a four function calculator](#) to enter and read displayed numbers using place value, to check their own answers to mental and written computations, and to add and subtract numbers beyond their mental and written computational ability
- engage students in describing simple fractions in relation to objects and collections ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$) and
- teach students to read, record and order money amounts and [carry out simple money calculations](#)
- engage students in identifying and representing surfaces, planes, corners, boundaries and a range of [2-D and 3-D shapes](#) and objects and tasks to classify shapes according to different features
- engage students in identifying and using [symmetry, asymmetry](#), congruence and [transformation of shapes](#) to complete pictures or patterns and teach location as a relative position (including left and right and on simple maps)
- teach students the concepts of volume, mass, time, weight and temperature and to identify and use formal units for measurement (time, capacity, length) and [use informal units](#) to quantify mass
- teach students to recognise key elements and patterns of time and to tell the time ([reading the hour](#) and [reading the half hour](#)) on analogue and digital clocks
- teach students to [predict outcomes of chance events](#) and develop the use of qualitative terms and in tasks to collect simple categorical and numerical data (count of frequency) and present this data [using pictographs and simple bar graphs](#)

For students achieving above the expected level, teachers should refer to the appropriate [VELS standards and progression points](#).

For students needing additional assistance, teachers should refer to the [Victorian Early Years Learning and Development Framework: Birth to 8 years](#) and [VELS level 1 standards and progression points](#).

Assessment

Effective teachers continuously monitor and track the progress of individual students.

At the beginning of the year, effective teachers:

- use data including VELS teacher judgements and the Mathematics Online Interview to understand the starting point for each student
- administer and analyse the [Mathematics Online Interview](#). Refer to the [Observation Notes](#) and 'Starting Points' to establish which question to use when recommencing the interview.

Throughout the year, effective teachers:

- schedule and document ongoing assessment to track individual student's progress
- provide students with regular opportunities for self assessment and self reflection
- administer and analyse the Assessment for Common Misunderstandings for identified students. These assessment tasks address key areas of number:
 - [LEVEL 2 – Place-value, the importance of moving beyond counting by ones, the structure of the base 10 numeration system](#)
 - [LEVEL 1 – Trusting the Count, developing flexible mental objects for the numbers 0 to 10](#)
- use the [VCAA Mathematics assessment maps](#) to help moderate student work
- provide timely and frequent feedback on assessment tasks.

At the end of each semester, effective teachers:

- integrate evidence collected throughout the semester to make on-balance judgements against VELS mathematics standards and progression points.

Planning and instruction

Assessment data is the starting point for curriculum planning and differentiated instruction.

Effective teachers:

- demonstrate capabilities as described in the [e⁵ Instructional Model](#)
- dedicate a daily one hour numeracy block (as a minimum) to explicit teaching, focusing on important numeracy ideas and making the focus clear to students
- use a range of flexible student groupings to provide the appropriate level of differentiated teacher support, including whole class focus, small groups, independent activities and whole class reflection and analysis
- provide independent time so students can practise what they know and to act on reflection and feedback, giving opportunities to make knowledge and skills automatic
- develop mathematical language by explicitly introducing new terms and symbols and expecting and encouraging correct use, making connections between language, symbols and materials
- provide opportunities and resources for students to manipulate concrete materials
- structure purposeful, authentic tasks that allow different possibilities, strategies and products to emerge and encourage higher order thinking skills
- develop numeracy understanding through strategic questioning and feedback by the teacher and explanation of reasoning and methods by the student

Recommended teaching strategies

Effective teachers:

- explicitly teach students [strategies to approach problems](#)
- select appropriate teaching strategies after analysis of the [Mathematics Online Interview](#)
- engage students in discussion, reflection and active construction throughout sessions to extend their thinking by building on their contributions and questions and to resolve misconceptions
- use a range of practices selected from the [twelve scaffolding practices](#). These are used throughout the five phases of instruction as described in the [e⁵ Instructional Model](#).

Selecting materials

Effective teachers select a range of materials that are:

- based on the students' developmental stages and the mathematics being explored
- appropriate to the learning context
- socially and culturally inclusive.

Years 3 and 4

Teacher knowledge

Effective teachers require:

- thorough knowledge of VELS mathematics domain [learning focus statements and standards](#) and progression points – levels [2](#), [3](#) and [4](#) to support planning for differentiated teaching
- knowledge of the [Mathematics Developmental Continuum P–10](#), the indicators of progress, teaching strategies and activities within the Continuum
- knowledge of key mathematical concepts in levels 1–4 of the [Developmental Overviews](#)
- knowledge of the [points of growth](#) in the Mathematics Online Interview
- awareness of the [Big Ideas Linked to the Fractions and Decimals Interview](#)
- knowledge of the particular needs that students may have in relation to English language and numeracy, including students from [Koorie](#), [ESL](#) and or [low SES](#) backgrounds.

Numeracy focus

Effective teachers determine the numeracy focus by referencing the VELS mathematics standards and progression points.

For students achieving at the expected level, effective teachers:

- teach the structure and place value of [whole numbers up to five digits](#), to [skip count](#) to and from various starting points using multiples (2, 3, 4, 5, 10, 100) and to round numbers up and down to the nearest unit (10, 100, 1000)
- engage students in representing, finding, comparing, ordering, adding and subtracting [simple fractions](#) and [decimals](#) to two decimal places [using physical models](#) and multiple representations
- teach the distributive property of multiplication over addition and engage students in tasks to use [multiplicative strategies](#)
- build students' understanding of addition, subtraction, [multiplication](#) and [division](#) to devise, describe, extend and test number patterns and sequences of [decimal numbers](#) generated using multiplication or division by 10, leading to the use of rules
- teach place value ideas and the properties of numbers and operations to assist mental computations and problem solving strategies (including [guess-check-improve](#)) and to check the reasonableness of the results of written and calculator computations by estimating the result
- teach the [application of all four operations](#) to solve and record problems involving whole numbers, [simple common fractions](#) (tenths and hundredths) and money and engage students in tasks to apply number skills to everyday contexts such as shopping, with appropriate rounding to the nearest five cents
- teach students to recognise that [sharing into equal-sized parts](#) (division) frequently leaves a remainder and to use number properties increasingly automatically, as they shuffle, split and combine numbers in ways that make calculation easier
- teach the [meaning of the equals sign](#) (=) as equivalence and as the result of a computation and engage students in tasks to write and interpret [number sentences](#) to describe real world situations of increasing complexity
- engage students in recognising, describing and representing [properties](#) (straight, curved, diagonal, horizontal and vertical lines) and to identify, label and measure common angles
- engage students in using nets to make 3-D shapes and counting edges, faces and vertices and copying and making simple patterns that involve translating, rotating and reflecting multiple copies of a polygon (closed shape with three or more angles and sides)
- engage students in estimating and measuring length, [area](#), volume, capacity, mass and time using appropriate instruments and to accurately [use formal units](#) to estimate and measure length and mass
- teach students to [read analogue and digital clocks](#) at five minute intervals and interpret timetables and calendars and engage students in [using grid references](#) and compass points to interpret and describe location and direction
- engage students in conducting chance experiments that use the concept of [fairness in events](#) and teach the experimental estimates of probabilities (moving from the concrete to the abstract) and how to display the results
- teach the recognition of different types of data including non-numerical (categories), separate numbers (discrete) or points on an unbroken number line (continuous)
- engage students in using all possible outcomes of a simple chance event and recognise samples as subsets of a population and to use lists, Venn diagrams and grids to show the possible combinations of two attributes.

For students achieving above the expected level, teachers should refer to the appropriate [VELS standards and progression points](#).

For students needing additional assistance, teachers should refer to the [Victorian Early Years Learning and Development Framework: Birth to 8 years](#) and earlier [VELS standards and progression points](#).

Assessment

Effective teachers continuously monitor and track the progress of individual students.

At the beginning of the year, effective teachers:

- use data including VELs teacher judgements, VCAA on demand testing, Mathematics Online Interview and Year 3 NAPLAN results to understand the starting point for each student
- administer and analyse the [Mathematics Online Interview](#) for students who require additional support. Refer to the [Observation Notes](#) and 'Starting Points' to ascertain the appropriate question with which to recommence interviewing
- administer and analyse [VCAA On Demand Adaptive Testing](#) – a range of computer-based assessments which identify a student's achievement level (use this tool to identify the spread of achievement within the class and then to use the progress test to gain more detailed information about individual students).

Throughout the year, effective teachers:

- schedule and document ongoing assessment to track individual student's progress
- provide students with regular opportunities for self assessment and self reflection
- administer and analyse [VCAA On Demand Progress Tests](#), which are linear tests where students receive a fixed set of questions and all students are presented with the same questions in the same order during the test designed to measure outcomes
- administer and analyse the Assessment for Common Misunderstandings for identified students. These tasks address key areas of number:
 - [LEVEL 3 – Multiplicative thinking, the key to understanding rational number and developing efficient mental and written computation strategies in later years](#)
 - [LEVEL 2 – Place-value, the importance of moving beyond counting by ones, the structure of the base 10 numeration system](#)
 - [LEVEL 1 – Trusting the Count, developing flexible mental objects for the numbers 0 to 10](#)
- regularly observe and document students' progress when working mathematically across all dimensions within the class context
- use the [VCAA Mathematics assessment maps](#) to help moderate student work using the VELs to support consistent, on-balance judgments about student achievement
- provide timely and frequent feedback on assessment tasks.

At the end of each semester, effective teachers:

- integrate evidence collected throughout the semester to make on-balance judgements against VELs mathematics standards and progression points.

Planning and instruction

Assessment data is the starting point for curriculum planning and differentiated instruction.

Effective teachers:

- demonstrate capabilities as described in the [e⁵ Instructional Model](#)
- dedicate a daily one hour numeracy block (as a minimum) to explicit teaching, focusing on important numeracy ideas and making the focus clear to students
- use a range of flexible student groupings to provide the appropriate level of differentiated teacher support, including whole class focus, small groups, independent activities and whole class reflection and analysis
- provide independent time so students can practise what they know and to act on reflection and feedback, giving opportunities to make knowledge and skills automatic
- develop mathematical language by explicitly introducing new terms and symbols and expecting and encouraging correct use, making connections between language, symbols and materials
- provide opportunities and resources for students to manipulate concrete materials
- structure purposeful, authentic numeracy tasks that allow different possibilities, strategies and products to emerge and encourage higher order thinking skills
- develop numeracy understanding through strategic questioning and feedback by teachers and explanation of reasoning and methods by students.

Recommended teaching strategies

Effective teachers:

- explicitly teach students [strategies to approach problems](#)
- select appropriate teaching strategies after analysis of the [Mathematics Online Interview](#)
- engage students in discussion, reflection and active construction throughout sessions to extend their thinking by building on their contributions and questions and to resolve misconceptions
- use a range of practices selected from the [twelve scaffolding practices](#). These are used throughout the five phases of instruction as described in the [e⁵ Instructional Model](#).

Selecting materials

Effective teachers select a range of materials that are:

- based on the students' developmental stages and the mathematics being explored
- appropriate to the learning context
- socially and culturally inclusive.

Years 5 and 6

Teacher knowledge

Effective teachers require:

- thorough knowledge of VELS mathematics domain [learning focus statements and standards](#) and progression points – levels [3](#), [4](#) and [5](#) to support planning for differentiated teaching
- knowledge of the [Mathematics Developmental Continuum P–10](#), the indicators of progress, teaching strategies and activities within the Continuum
- knowledge of key mathematical concepts in levels 2–5 of the [Developmental Overviews](#)
- knowledge of the [points of growth](#) in the Mathematics Online Interview
- knowledge of the [Big Ideas Linked to the Fractions and Decimals Interview](#)
- knowledge of the particular needs that students may have in relation to English language and numeracy, including students from [Koorie](#), [ESL](#) and or [low SES](#) backgrounds.

Numeracy focus

Effective teachers determine the numeracy focus by referencing the VELS mathematics standards and progression points.

For students achieving at the expected level, effective teachers:

- engage students in exploring differences and similarities in size and order of small (to one thousandth) and [large \(to 1,000,000\) numbers](#) and the use of decimals, ratios and percentages to find equivalent [representations of common fractions](#)
- teach integers (positive and negative whole numbers and zero), [common fractions](#) and decimals; square, prime and composite numbers and how to calculate simple powers of whole numbers
- teach students [how to use factors](#) for [strategies for multiplication](#), creating factors sets for [mental strategies for division](#), how to identify the lowest common factor for two or more numbers to [create sets of multiples](#) and how to find the lowest common multiple and the factors of a given number using systematic methods such as arrays
- engage students in tasks to use [mental](#) and written algorithms for the [four operations](#) for natural numbers (whole positive numbers) and [decimals](#) (to two decimal places) and apply these in practical contexts, including money and teach the [simple recursion rule and a function rule](#) to describe, create or extend a simple number sequence
- engage students in identifying and using [arithmetic relationships within number sentences](#) to solve problems without calculating and teach a repertoire of strategies – guess-guess-check (systematic trial and error), logical arithmetic reasoning and inverse operations to solve a wider range of [number sentences](#)
- engage students in identifying [relationships between variables](#) – to describe them with words and symbols and form simple equations, to verbally describe straightforward relationships between sets of everyday and mathematical objects and to illustrate relationships with [Venn diagrams](#) and two-way tables (Karnaugh maps) as appropriate
- teach estimation for computation and how to apply criteria to determine the [reasonableness of estimates](#) and how to develop and test conjectures, recognising that a single [counter-example](#) is sufficient to invalidate a conjecture
- engage students in tasks to [classify shapes](#) and solids using the properties of [lines](#), [angles](#) and surfaces and teach the use of scale to [enlarge and reduce a shape](#) and to estimate and measure [time](#), capacity, angles, [perimeter and area](#) and to [convert measurements](#) between [metric units](#)
- engage students in exploring relative location using size, scale and direction and tasks that show and investigate relationships and connections using [network diagrams](#)
- engage students in tasks to describe probabilities of chance outcomes using words and fractions and decimals between 0 and 1 and teach the experimental estimates of probabilities (moving from the concrete to the abstract)
- teach students to recognise and distinguish between different data types (discrete and continuous), represent data in [appropriate graphical displays](#) and calculate and interpret mean, [median](#) and mode for data
- engage students in tasks to recognise and investigate the [usefulness of numeracy](#) in [real world situations](#) and through history.

For students achieving above the expected level, teachers should refer to the appropriate [VELS standards and progression points](#).

For students needing additional assistance, teachers should refer to earlier [VELS standards and progression points](#).

Assessment

Effective teachers continuously monitor and track the progress of individual students.

At the beginning of the year, effective teachers:

- use data including VELS teacher judgements, VCAA on demand testing and Year 5 NAPLAN results to understand the starting point for each student
- administer and analyse [Fractions and Decimals Online Interview](#)
- administer the assessment materials to assess student's multiplicative thinking ([Scaffolding Numeracy in the Middle Years](#)) and analyse the assessment outcomes using the [Learning and Assessment Framework for Multiplicative Thinking \(LAF\)](#) and plan for future learning by using [learning plans](#).
- administer and analyse [VCAA On Demand Adaptive Testing](#) – a range of computer-based assessments which identify a student's achievement level (use this tool to identify the spread of achievement within the class and then to use the progress test to gain more detailed information about individual students).

Throughout the year, effective teachers:

- schedule and document ongoing assessment to track individual student's progress
- provide students with regular opportunities for self assessment and self reflection
- administer and analyse [VCAA On Demand Progress tests](#), which are linear tests where students receive a fixed set of questions and all students are presented with the same questions in the same order during the test designed to measure outcomes
- administer and analyse the Assessment for Common Misunderstandings for identified students. These assessment tasks address key areas of number.
 - [LEVEL 4 – Partitioning, the missing link in building common fraction and decimal knowledge and confidence](#)
 - [LEVEL 3 – Multiplicative thinking, the key to understanding rational number and developing efficient mental and written computation strategies in later years](#)
 - [LEVEL 2 – Place-value, the importance of moving beyond counting by ones, the structure of the base 10 numeration system](#)
- regularly observe and document students' progress when working mathematically across all dimensions within the class context
- use the [VCAA Mathematics assessment maps](#) to help moderate student work using the VELS to support consistent, on-balance judgments about student achievement
- provide timely and frequent feedback on assessment tasks.

At the end of each semester, effective teachers:

- integrate evidence collected throughout the semester to make on-balance judgements against VELS mathematics standards and progression points.

Planning and instruction

Assessment data is the starting point for curriculum planning and differentiated instruction.

Effective teachers:

- demonstrate capabilities as described in the [e⁵ Instructional Model](#)
- dedicate a daily one hour numeracy block (as a minimum) to explicit teaching targeting important mathematical ideas and making the focus clear to students
- use a range of flexible student groupings to provide the appropriate level of differentiated teacher support, including whole class focus, small groups, independent activities and whole class reflection and analysis
- provide independent time so students can practise what they know and to act on reflection and feedback, giving opportunities to make knowledge and skills automatic
- develop mathematical language by explicitly introducing new terms and symbols and expecting and encouraging correct use, making connections between language, symbols and materials
- provide opportunities and resources for students to manipulate concrete materials
- structure purposeful, authentic numeracy tasks that allow different possibilities, strategies and products to emerge and encourage higher order thinking skills
- develop numeracy understanding through strategic questioning and feedback by the teacher and explanation of reasoning and methods by the student

Recommended teaching strategies

Effective teachers:

- explicitly teach students [strategies to approach problems](#)
- select appropriate teaching strategies including [classroom activities](#) after analysis of the Fractions and Decimal Online Interview
- engage students in discussion, reflection and active construction throughout sessions to extend their thinking by building on their contributions and questions and to resolve misconceptions
- use a range of practices selected from the [twelve scaffolding practices](#). These are used throughout the five phases of instruction as described in the [e⁵ Instructional Model](#).

Selecting materials

Effective teachers select a range of materials that are:

- based on the students' developmental stages and the mathematics being explored
- appropriate to the learning context
- socially and culturally inclusive.

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